
Plate.

[Read before the Otago Institute, 3rd October, 1876.]

In introducing to your notice such a dry subject as that of this paper, my excuse must be, that in these days of great public works in the colony, its consideration may prove useful, more particularly as the best authorities, such as "Buck, on Oblique Bridges," cannot be got at all here; and another thing is that, being confessedly a difficult subject, it may be found all the more deserving of some attention, at any rate from engineers. But before describing particularly the details of the design of a skew arch, it may not be altogether uninteresting to look for a little at what is known of arches in general, built during past ages of the world, and also at the position of bridges in Otago at the present day.

History of Arches.

Arches of masonry—that is, ordinary arches—are of very ancient origin. The earliest known remains are to be found in the ruins of Nineveh; they occur next among the oldest buildings of ancient Egypt. The caves of Adjunta and Ellora in British India reveal to this day circular and pointed arches in their subterranean temples, shewing the skill and handicraft of a race of men who lived probably three thousand years ago. Dr. Robinson, the American traveller, discovered at Jerusalem, part of a very old arch, which appears to have connected Soloman's Temple with a portion of the city, from which it was separated by a small valley or gorge. He found only a few stones still in position at the springing of the arch next the temple, or what is now the mosque, but these were of very great dimensions. From measurements made across the valley, the span of this arch was found to be about 350 feet. I have calculated the weight of one of these arch stones from such dimensions as are given, assuming the material to be limestone, and it turns out to be twenty tons! The Romans next appear to have availed themselves of the principle of the arch, as may be seen in their domes, bridges, vaults, and sewers. And lastly, in the explorations which were made forty years ago in Mexico, together with pyramids and ancient cities, several specimens of arches also came to light, which are believed to be at least two thousand years old. It thus appears that the principle of the arch was known and appreciated from the earliest ages. In modern times the construction of arches has received greater attention, but we have no knowledge of the existence in ancient or modern times of a skew arch, until 1580, when one was built in Italy. In our own country, no attempt was made to erect such a structure, until the introduction of railways com-

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peled engineers to avail themselves of the principle, and now skew arches are to be met with on nearly every line of railway in Great Britain.

Bridges in Otago.

In a colony like this, bridges as a rule have hitherto been constructed of timber. This is owing to the rapidity and cheapness with which such works can be carried out, and the circumstances of a young colony seem to justify such economy. But I question whether there is in this any real economy in the long run. For even where the workmanship is of the best and the quality of the material unexceptionable, timber bridges require constant repairing, and cannot at most be expected to last more than fifteen or twenty years. Iron bridges are no doubt very convenient in railway works and for great spans, but even they are not equal in durability to good stone arches. This fact has caused the Parisians to substitute arches of masonry in their main thoroughfares, for the iron bridges previously erected there. English engineers are coming round also to admit, that stone after all, has many advantages over iron in permanent structures. Most of the bridges recently built in Otago by the Provincial Government are a great improvement on timber pile bridges, as the piers and abutments have been built in masonry, the superstructures only being in timber. The only stone arches however, of any importance, which Otago at present possesses, are those on the Main North Road from Dunedin to Oamaru, five in number, and two or three on the road from Palmerston to the interior; the greatest span of any one of these being sixty feet. There are no skew arches in Otago and I am not aware of the existence of one in any part of New Zealand at present.

On bridges in Otago there was spent by the Provincial Government, during last year alone, no less than £34,751, so the kind of material which is best for their construction is a consideration worthy of the attention of all good colonists.

Skew Arches.

I will now proceed to describe the principles and construction of skew arches in masonry, illustrated by drawings, and a small model of an actual segmental arch, built seven years ago. These drawings are No. 1, and No. 2, appended to this paper: No. 1 shewing half elevation, plan and section; and No. 2, the development, on a flat surface, of the intrados and extrados, which is the more important and necessary drawing for the guidance of the builder.

Definition.

A skew arch in masonry may be defined as being an arch whose abutments are equal, parallel, but not directly opposite each other, and whose courses, from face to face, are not parallel but inclined to abutments, and are spiral surfaces. The angle of skew, θ on Drawing No. 1, is the angle
contained between the face line and its position, if drawn at right angles to the abutment. Most writers give the complement of this, or $BA\,D$, as the angle of skew, but this does not appear appropriate; and Professor Rankine, I find, gives an angle of equal value to the one I have adopted. The spiral surfaces are the beds formed by the continuous courses of arch stones, or "voûtiers," which run from one face of the arch to the other. One such surface is shown in Drawing No. 2, where $cine$ is half of a hollow cylinder, and $dy$ (as shaded) represents a spiral course, or a "coursing spiral," as it is called. It may be regarded as described by the revolution of radius $yo$ through angle $(yol-d\,rp)$, simultaneously with its progression from the position $dr$ to $yo$ along the axis of the cylinder $ro$. The peculiar direction of the coursing spirals is necessary to bring these courses as nearly as possible at right angles to line of pressures. Beds running at right angles to these are heading spirals, and in the actual arch are the end surfaces of arch stones, where these abut on one another. The best guide to show the curious lines, angles, and surfaces which result from a bridge being built on the skew principle, is a model; this simplifies many difficulties.

$\textit{Stability.}$

It is not properly within the scope of this paper to go into the equilibrium of arches. But besides what I have unavoidably stated, that the coursing spirals should be at right angles to the line of pressures, I may add that the general principles of stability are the same as in ordinary arches, the chief one of which I shall only mention. Taking half the arch—the half-arch ring must carry its own weight, that of the spandril wall above it, with its load, acting vertically through the centre of gravity of the mass, and be supported by two pressures, one upward and inclined from the abutment (see Fig. 1) and the other horizontal, passing through the keystone; these pressures should intersect each other in the line of gravity of the mass. It is not usual, unless arches are of great span, to investigate by actual calculation the line of pressures; the ordinary formulae in use are sufficient for determining the thickness of abutments and arch ring.

$\textit{Design.}$

In designing a skew arch a ground plan $AB\,CD$, as shown in drawing No. 2, is prepared provisionally from measurements made at the site of the bridge. Afterwards, as will appear, it may have to be slightly altered before finally fixing its dimensions.
Transactions.—Miscellaneous.

Section on Square and Skews.

A section on the square, of the arch with given rise and span, is then described as segment $AB$. A series of ordinates are drawn parallel to abutments from the chord $Ab$ on plan, to the solid face line $AB$, and continued where necessary to the extrados of section on square. From the points where these lines meet $AB$, ordinates are then drawn at right angles to $AB$, and on these heights are carefully marked corresponding to the heights measured between the chord $Ab$ and the two surfaces of section on square, or its intrados and extrados. By this means, the elevation on skew or of face, can be accurately drawn; the curved lines of this elevation are portions of ellipses.

Intrados.

The development of the intrados, or sofit of the arch, must next be designed. $ABCD$ on left of plan, represents the hollow surface or underside of arch, turned backwards round the impost line $AC$ and spread out as a flat surface. It is drawn in this way:—Through $A$ produce chord $Ab$, making its length equal length of segment. Draw $bB$ at right angles to segmental length $Ab$ from $b$ and equal to $bB$ on plan, and join $AB$ as shewn by dotted lines; that line is the development of a heading spiral. Complete the figure by drawing $BD$, $DC$, parallel and equal to the sides opposite to them. Draw $AE$ from $A$ at right angles to heading spirals $AB$ or $CD$, and produce $BD$ to meet it in $E$, and complete figure $AbEF$, making $AFE$ a right angle. The length $bE$ or $AE$ is readily calculated thus:—$bE = \frac{bB}{\sin \theta}$. Angle $EAF$ or $\beta$ is the angle of intrados, which in this arch is $21^\circ 3'$; and $\tan \frac{\beta}{2} = \frac{Bp}{Fp}$. The number of courses of arch stones, always an odd number (in this case 27), is decided on and laid off along the heading spiral $CD$. If $AE$ does not coincide at its intersection with $CD$ with one of these divisions, it must be made to do so—in other words, the development requires adjustment. This is effected in several ways; such as altering the number of divisions of face line, altering span, radius, or length of impost. In Bathlin Bridge, it was done by ruling the line $AE$ through the nearest face joint, and altering impost line $AC$ to correspond; thus avoiding any disturbance of angles or number of courses. On $AC$, mark off as many skewbacks as there are courses between $C$ and $AE$, and through these points draw lines parallel to $AE$; these lines will intersect the courses marked on the heading spiral between $C$ and $AE$, and will represent coursing spirals. Complete drawing of courses through remaining divisions on $CD$, and draw development of face line $AB$ by ordinates laid down at right angles to $Ab$ at distances equal to those between the ordinates along the segment or section on square, and of lengths equal to those of the corresponding ordinates between the chord $Ab$ and face line.
Arthur.—On Skew Arches.

AB. In the same way draw face line CD, and then the heading joints of skewbacks, and face quoins at right angles to coursing joints. The other heading joints of soffit may be left to the builder to arrange, care being taken that a good bond is maintained throughout. This is the usual method of designing the development of the intrados, and, taken in connection with plan and development of extrados on same drawing, it is very convenient, so far as showing the relation of the different parts. But it is reversed from its normal position, or turned upside down: this will be referred to again.

**Face Joints.**

The elevation on skew or elevation of face, must have the face joints, or rather their chords, drawn from a centre lower than the axis of the cylinder and from which point they radiate. This property in skew arches was discovered by Mr. Buck, and is known as the focal eccentricity, or the difference between centre of cylinder and centre from which chords of face-joints radiate. The formula for calculating this is $60 = \text{Rad.} \times \tan \theta \times \tan \phi$; and a geometrical method is given by diagram in "Masonry and Stone-cutting," by Dobson.

**Extrados.**

The development of the extrados should next be drawn. This is done similarly to that of the intrados, but with this difference, that the coursing spirals are not at right angles to the heading spirals. The reason of this is that the length of the coursing spirals, measured or projected on the axis of the cylinder is the same for intrados and extrados, but the length of the heading spirals is greater in the extrados than the intrados. Another consequence is that the angle of extrados $G \, A' \, H$ or $\phi$, is greater than the angle of intrados $\beta$. The figure $A' \, B' \, C' \, D'$, Drawing No. 2, represents a development of the extrados.

**Elevation, etc.**

The more essential designs having now been completed, from these the elevation, plan and cross section, shewn in Drawing No. 1, are drawn. In the elevation, the only matters further to be noted are that the back of the arch-stones should be stopped, and the courses of spandril walls built to correspond, thus securing greater stability than if these precautions were neglected. The plan shews the peculiar serrated appearance which the springers or skewbacks present, owing to the obliquity of the courses; and it will also be perceived that the boss stones at the obtuse and acute angles, are greater and less respectively than the other skewbacks, for the same reason.

**Bathlin Burn Bridge.**

The remaining peculiarities of skew arches, and the rules which are
necessary to be observed in construction, may perhaps be more conveniently laid before you if, during the rest of this paper, I adhere as nearly as possible to a description of the methods actually adopted in the Bathlin Bridge. Drawing No. 1 shows the ordinary designs that were used, and the principles above explained and represented by Drawing No. 2, were carried out in a modified form.

Skewbacks or Springers.

First, the skewbacks were cut to sizes, and templets taken from impost line $AC$ (as $def$ templet for soffit), and built into position, the face of quoins being cut to lie in perpendicular plane; the stones dressed to thickness of arch-ring, and the extrados left rough and quarry-faced—templet, $a \ b \ \ y \ h$, not being used. When skewbacks were fixed, centreing and laggings were erected carefully. The intrados was drawn on paper on a scale of two feet to an inch, but instead of being turned upside down, as in drawing No. 2, it was designed in its normal position; an elevation on skew and section on square, on same scale, and a full-sized drawing of twisting rules, as sketched in No. 2, were made; but no development of extrados was drawn, it was found sufficient to calculate angle of extrados $\phi$. In using the normal development of intrados for this arch, it was found to secure much convenience to the builder, who was thus saved the awkwardness of always treating the arch-stones as inverted. A large wooden platform was constructed adjoining site of bridge, and on it a full-size drawing was made of development of intrados in its natural position, and the sizes and joints of all the stones arranged on it. On the laggings the coursing spirals were laid down or marked off from the development on the platform.

Angle of Twist.

The angle of twist ($\phi-\beta$) having been found by calculation, the winding strip and parallel rule were drawn full size, and from these the actual rules were cut, which the builder used in dressing the beds of the arch-stones to the proper twist. The divergent portion of winding strip shown is that applicable to stones of three feet in length, applied at right angles to coursing spirals. These twisting rules may be used either as rules or templets.

Face angle of Quoins.

The next thing in importance, requiring great care, was the working the faces of the quoins to the proper angle with the soffit. From the springing to the keystone the faces of quoins make different angles with the coursing joints. On the side next the acute angle these angles are acute, and on the side next the obtuse angle they are obtuse; and the angles equi-distant from keystone are complements of one another. The angle in each case was
measured on the laggings between a perpendicular plane in line of face and
the coursing spiral at the particular point. A convenient method to find this
angle is with a board, one edge of which is dressed to the curve of the
soffit in the direction of spiral courses, applied at a point marked at the
centre, or any convenient point, of this
curved edge to the face-line on laggings,
and resting on laggings in the true
position of a coursing spiral. A plumb-
line run then carefully along the upper
dge of board till the weight points
exactly on the face line, will give a point
on that edge, which joined by a pencil
line with centre-point on curved edge below, will represent the actual angle
and its complement at the particular course where the measurement is
taken. From this board the templets may be made.

The obliquity of the arch causes a curious result in the length of the
top and bottom of quoins. That is, on the acute side of arch the soffit of
the quoins is longer than the top or extrados, while on the obtuse side the
soffit of the quoins is shorter than the extrados.

Working of Arch Stones.

In the operation of dressing or working the arch stones, such as the
quoins, a plain surface is first prepared approximating in size and direction
a coursing joint or bed, and on this the curve of the arch soffit, in the same
direction, is drafted. The twisting rules are next applied, and the plane
surface is worked off to the proper twist, right handed or left, as the case
may be. Then the soffit is worked to the arch square—(a rule made by
joining a straight-edge at right angles to a rule having the curve of the arch)—
the heading joint is dressed at right angles to soffit, and the face to a rule
or templet forming the correct angle of the face. There are also the neces-
sary dimensions of quoins to be taken from diagram on platform and from
drawings as required.

Though not all mathematically accurate, these rules as I have now
given them, for the building of a skew arch, are those commonly followed
in practice, and give sufficiently good results to justify their use.

Skew Arches of other forms.

In the above description of the design and construction of a skew arch,
it will be observed that a segmental arch only has been treated of. There are
other forms of skew arches rendered sometimes unavoidable by the peculiar
circumstances of the case, such as an elliptical skew arch, and a skew arch
the plan of which is on a curve.
As to elliptical skew arches, the principles involved in designing these are the very same as for segmental arches; therefore, in applying these, the ellipse may be divided into two or more portions, which may be taken as continuous segments of circles, or an approximate segment of a circle in position and length may be assumed as the actual one, and developed accordingly.

And as to skew arches on curves, these may be designed as if the faces were straight, in the same manner as given in this paper, and the correction in the lengths of courses applied afterwards to the development of the intrados, lengthening those on the convex side and shortening those on the concave side. But it is evident that the principle can only be applied safely within certain limits, and in so doing, it must always be borne in mind that the properties of the lines and sections of a cylinder lie at the basis of the design of any skew arch.

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Among the most remarkable speculations of the present century is the speculation that the axioms of geometry may be only approximately true, and that the actual properties of space may be somewhat different from those which we are in the habit of ascribing to it. It was Lobatchewsky who first worked out the conception of a space in which some of the ordinary laws of geometry should no longer hold good. Among the axioms which lie at the foundation of the Euclidian scheme he assumed all to be true except the one which relates to parallel straight lines. An equivalent form of this axiom, and the one now generally employed in works on geometry, is the statement that it is impossible to draw more than one straight line parallel to a given straight line through a given point outside it. In other words, if we take a fixed straight line $A B$, prolonged infinitely in both directions, and a fixed point $P$ outside it; then, if a second straight line, also infinitely prolonged in both directions, be made to rotate about $P$, there is only one position in which it will not intersect $A B$. Now Lobatchewsky made the supposition that this axiom should be untrue, and that there should be a finite angle through which the rotating line might be turned, without ever intersecting the fixed straight line $A B$. And in following out the consequences of this assumption, he was never brought into collision with any of the other axioms, but was able