concise order to go egg-hunting and not waste his time that way."

I do not propose to treat this subject from a scientific point of view; but the bones and ovens I saw at Waingongoro in 1866, and the evidence obtained by the Hon. Walter Mantell in 1848, at Awamoa, certainly afford proofs that the moa lived down to very recent times.

Art. LIX.—*On the Mechanical Description of a Straight Line by means of Link-work.*

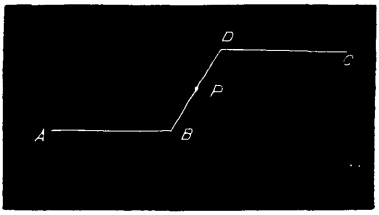
By W. Steadman Aldis.

[*Read before the Auckland Institute, 22nd October, 1888.*]

Those who are familiar with the early history of the steam-engine will remember that in the first form, known as Newcomen's, the pressure on the piston was only employed to pull the beam down; and that thus an attachment of the piston to the beam by means of a chain passing over a circular head was sufficient to insure the proper motion. When, however, Watt closed in the cylinder and drove the piston both up and down by steam-pressure, it became necessary to connect the rectilinear motion of the piston-rod with the circular motion of the end of the beam in a manner which should enable the piston-rod to exert a push as well as a pull on the beam.

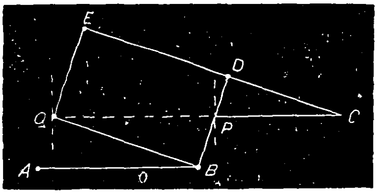
The geometrical problem was to discover a means of making one point move in a straight line while connected by rigid bars with another point moving in a circle.

The solution adopted by Watt was an approximate one, and depended on the following geometrical proposition:—



Let AB and CD be two fixed rods capable of turning in the same plane about the points A and C, while their other ends are connected by a bar, which is hinged to them at B and D respectively. Then, if this arrangement of bars be made to assume all possible positions, any point, P, in the connecting bar will describe a curve called a lemniscoid, of the general shape of an elongated figure of eight. At the point of crossing of the two branches a portion of either is very approximately a straight line, and thus if the rods AB and CD do not turn through *too great* an angle, P may be attached to a piston- or slidevalve-rod, which is constrained to move in a straight line, without danger of breaking the machinery.

The arrangement thus suggested would require a greater space for the machinery than is ordinarily available if CD represented the half of the beam and P the point of attachment of the piston-rod. By means of an arrangement of parallel rods the *motion* of P is multiplied, so to speak, in the following manner:—



For simplicity's sake, suppose that AB and CD are equal, and P the middle point of BD. Imagine CD to be produced to E, making DE equal to CD, and let two other rods, as EQ and QB, equal to BD and DE respectively, be hinged to the others at E and B, and to each other at Q.

Then elementary geometry shows that throughout the motion DEQB is always a parallelogram, and, since QE is double of DP, and CE double of CD, the points C, P, Q are in a straight line, and CQ is always double of CP. Hence the path described by Q must be similar to that of P on a scale twice as large, and, as P moves approximately in a straight line, so also will Q.

CE represents half the beam, and Q is the point of attachment of the piston-rod, while P serves as a point of attachment of a pump- or valve-rod.

These motions are illustrated by the models shown. The problem of connecting an exact rectilinear motion with a circular one has only been solved in comparatively recent times, the first arrangement of link-work effecting this object having been devised by M. Peaucellier, a French engineer officer, in 1864. Other methods of achieving the same result have since been discovered.

The geometrical theorem on which M. Peaucellier's apparatus depends is the following:—

If a fixed point, A, on the circumference of a circle be joined with any other point, P, and a length, AQ, be measured on AP such that the product of the lengths AP and AQ always has the same value, then, as P moves round the circle, Q will move on a straight line.

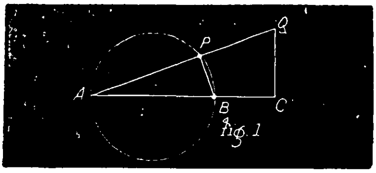


fig. 1

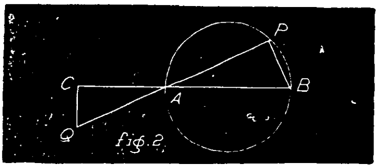


fig. 2

Let AB (figs. 1 and 2) be the diameter of the circle through A. Take AC so that the product of AB and AC is equal to the constant value of that of AP and AQ. Then C is a fixed and determinable point.

If then CQ and BP be joined, since AP AQ is equal to AB AC, it follows that PBCQ is a cyclic quadrilateral, and therefore the angle ACQ is equal to the angle APB—that is, is a right angle. Hence Q always lies on the straight line drawn through C at right angles to AC.

The Peaucellier cell, as the framework is called, consists of seven bars, four of which are of one length, two more are equal to one another, but unequal to the former, while the seventh may be taken arbitrarily. These are jointed together as in fig. 3, the four equal bars forming a rhombus, ABPD, the other two equal ones being attached to this at B and D,

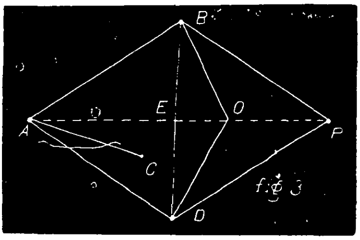


fig. 3

and to each other at O, while the seventh is attached to the rhombus at A.

The points C and O are fixed points, and the distance between them is equal to the length of the seventh bar, CA.

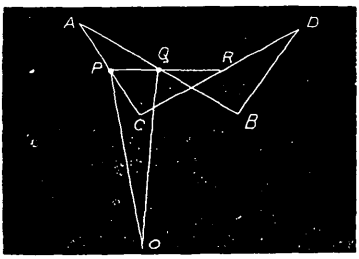
Elementary geometry shows that the points A, O, P will always lie in a straight line, and also that BD is at right angles to AP, and bisects it.

Hence (by Euclid, ii., v.) the rectangle AO OP is equal to the difference between the squares on AE and EO, or to the difference between the squares on AB and BO—that is, the rectangle AO OP has always the same value in whatever manner the bars may be turned round their hinges. But as they turn the point A moves on a circle whose centre is C and radius CA, and which therefore passes through O, since CO is equal to CA. Hence P must move on a straight line perpendicular to CO.

If the distance CO be taken different from the length of CA the point P will describe a portion of a circle.

A second method of producing a rectilinear motion by linkwork, depending on the same geometrical proposition, requires only six bars, equal in pairs.

Four of these are linked together so as to form an antiparallelogram, ABDC; AB, CD being equal bars, and also AC, BD. Then it is a consequence of elementary geometry



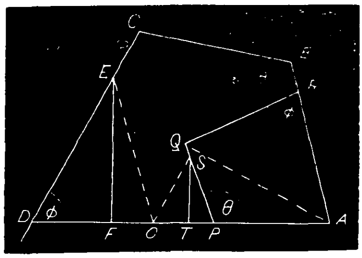
that, if P, Q, R be three points on the rods AC, AB, CD respectively such that PQR is parallel to AD or CB in any one position of the framework, they will always satisfy this condition in whatever way the bars be turned about their joints.

The remaining pair of bars are linked to AC and AB at P and Q, and to each other at O.

Then the ratio of PQ to BC is fixed throughout the motion, and also that of PR to AD. Hence the ratio of the product of PQ and PR to that of CB and AD is given. But this latter product is invariable (Euclid, vi., D). Hence, also, the former has always the same value.

Thus, if OP be fixed, since Q must describe a portion of a circle, with O as centre, passing through P, the point R will describe part of a straight line perpendicular to OP.

As the bar OP does not move it may be dispensed with, provided the points O and P are fixed at a distance from one another equal to OQ. Thus this mechanism requires only five bars, instead of the seven involved in the Peaucellier cell.



Another arrangement of links, by which a very long swing in a straight line can be obtained, requires some preliminary geometrical explanation.

Let ABCD be any quadrilateral, and APQR a second having its sides AP AR coincident in direction with AD and AB. Let also the lengths of the lines AP, PQ, QR, RA be in the same ratios to one another as the lengths AB, BC, CD, DA. Then it follows that the figures ABCD and APQR will be similar, and if the lines represent rods jointed at all the points of meeting the angles ARQ and ADC will always remain equal in whatever manner the rods are turned about their joints.

We shall now prove that, if S be any point in PQ, a point E can be found in DC such that when ST and EF are drawn perpendicular to AD the length FT will remain invariable in whatever manner the links are turned about their hinges.

Let the lengths of AB, BC, CD, DA be denoted by a, b, c, d, and those of AP, PQ, QR, RA by p, q, r, s. Also, let the angle ABC or APQ be called θ, and the angle ARQ or ADC be called ϕ.

Then, joining AQ, by a well-known trigonometrical theorem—

$$AQ^2 = p^2 + q^2 - 2pq\operatorname{Cos}\theta = r^2 + s^2 - 2rs\operatorname{Cos}\phi.$$

$$\therefore\ pq\operatorname{Cos}\theta - rs\operatorname{Cos}\phi = \tfrac{1}{2}(p^2 + q^2 - r^2 - s^2).$$

Let, now, PS $= x$, DE $= y$,

Then FD $= y\operatorname{Cos}\phi$, TP $= x\operatorname{Cos}(\pi - \theta) = -x\operatorname{Cos}\theta$.

$$\therefore\ \text{FT} = \text{PD} - \text{DF} - \text{TP}.$$

$$= d - p - y\operatorname{Cos}\phi + x\operatorname{Cos}\theta.$$

If, now, y be so chosen that $\frac{y}{x} = \frac{rs}{pq}$, we have

$$x\text{Cos}\theta - y\text{Cos}\phi = x\text{Cos}\theta - \frac{rs}{pq}x\text{Cos}\phi.$$
$$= \frac{x}{pq}(pq\text{Cos}\theta - rs\text{Cos}\phi).$$
$$= \frac{x\,(p^2+q^2-r^2-s^2)}{2pq}.$$

Hence, $\text{FT} = d - p + \frac{x}{2pq}(p^2+q^2-r^2-s^2)$, which is independent of the angles θ and ϕ, and retains therefore the same value, however the links be turned about their joints.

By taking S suitably, this value of FT may be made anything we please. If, for instance,—

$$\frac{x}{pq}(p^2+q^2-r^2-s^2) = -(d-p),$$
$$\text{or } x = \frac{pq(d-p)}{r^2+s^2-p^2-q^2},$$

$$\text{FT becomes } \tfrac{1}{2}(d-p) \text{ or } \tfrac{1}{2}\text{DP}.$$

Hence, if FO be taken equal to FD, OT will also equal TP. It follows that EO must equal ED, and SO must equal SP.

Thus, if to the original framework there be attached two other links at the points E and S, equal respectively to ED and SP, and these links be hinged together at O, the point O must always lie somewhere in the straight line AD.

Thus, if APD be fixed, and the other links be moved any possible way, the point O will describe the straight line DPA.

In the particular case which the model illustrates the points R and B coincide, so that $a = s$; also q and r are equal, and therefore also b and c. The figures being similar, we have in general—

$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{d}{s},$$

Hence, in this particular case $pd = as = s^2$, and

$$x = \frac{pqd - p^2q}{s^2-p^2} = \frac{(s^2-p^2)q}{s^2-p^2} = q.$$

Hence the point S must coincide with Q.

Also, $y = x\frac{rs}{pq} = x\frac{s}{p} = q\frac{a}{p} = b$; so that E coincides with C.
