The motion of the sun, as deduced from the proper motion of the stars, is, according to Proctor ("The Sun"), 150,000,000 miles per year—that is, 25,154·38 ft. per second, the line of motion being inclined to the earth's orbit at about 53° in longitude 285°. This is about 60° to the earth's axis. Resolving along the equatorial plane, this gives—

$$u = 21{,}784 \text{ ft. per second;}$$

and, as $\omega = 0{\cdot}000073$ rad. per second,

$$\text{we get } f = -1{\cdot}584 = -\frac{g}{20} \text{ (about) when } \theta = 0.$$

Similarly, this would be the acceleration along the radius at $\theta = 90°$; so that the weight of a body at the equator should vary by 10 per cent. every twelve hours. The motion of the earth in space, therefore, cannot be as great as deduced from the proper motions of stars.

If A be the angle through which a plumb-bob is deflected by this spacial acceleration, we have—

$$\tan. A = \frac{-u\omega \cos. \theta}{u\omega \sin. \theta \cos. \lambda + g}.$$

Perhaps this in some part reconciles the seismological tides found by Milne with Lord Kelvin's value of the rigidity of the earth.

From an experimental point of view the method is very accommodating. Being a harmonic quantity, it does not matter when we set our instruments, which may, for the same reason, measure variations in pressures. Being an acceleration, it may be magnified to any extent by using large masses. With sufficiently delicate apparatus, and observations extending over a long period, it might be possible to deduce the relative motion and distance of a star for which the earth's orbit failed to show any parallax.

Art. LII.—*Mathematical Treatment of the Problem of Production, Rent, Interest, and Wages.*

By Douglas Hector.

[*Read before the Wellington Philosophical Society, 11th February, 1902.*]

The following attempt at a mathematical treatment of some of the problems of political economy was not originally intended for publication, but I have been persuaded to submit it as a paper to the Wellington Philosophical Society. I have not solved all the interesting points in the subject, but merely a few of the more simple ones. Several attempts have been made to treat political economy mathematically, but they have chiefly resulted in failure, for the reason that the mathematics has taken quite a subordinate part, being used to express the result of elaborate reasoning by words. It is like the man who keeps a watchdog and does the barking himself.

The most successful attempt so far seems to have been made by Professor Jevons,* but he states in his preface that, although many of the problems might have been solved more directly, he preferred to limit himself to the simplest possible mathematics, thus the book hides rather than shows the value of applying mathematics to the subject. Another writer on the subject is Professor J. D. Everett.† A long list of other writers is given at the end of Professor Jevons's book, but the two mentioned are the only mathematical ones to which I have been able to refer; and, from a remark on the customary method of treatment in Professor Everett's paper, I believe that the proofs in the following paper are new, though the results have in many cases been previously obtained by a patient application of logic.

The fundamental principle which is assumed in the following is that in the serious affairs of life a person always endeavours to obtain the maximum return on an investment. This one might almost call an axiom, and as such it is used. With regard to the definitions, I have defined the quantities as I intend to use them, and as long as a definition and its use are consistent no more is required of it.

Many people think that the application of mathematics to political economy is an almost impossible proceeding. The science, they say, is too vague and conditional for it to be possible. The same might have been said of other sciences in their beginnings, but which have since had mathematics successfully applied to them. For instance, what is more capricious than evolution? yet Professor Pearson is successfully applying mathematics to this subject. The problems of political economy in many cases resemble problems in dynamics, and it is quite a possibility that its elements might be expressed in terms of energy which would thus bring it more into line with other branches of applied mathematics. In fact, so apparent are the advantages of the mathematical treatment of the subject to many that a well-known professor jokingly said, in a lecture on the representation of facts by curves, that before long we should probably see our legislators,

* "Theory of Political Economy."

† "On Geometrical Illustrations of the Theory of Rent" (Jour. R.S S., lxii., 703).

instead of preparing lengthy speeches, framing the laws of the country by means of squared paper and curves.

Some may demur to the latter part of the definition of interest as requiring proof, but it is rather a historical point than otherwise, and, in any case, does not affect results obtained.

Definitions.

1. "Production" is the changing of form, constitution, place, or time of a natural product of nature in order to render it efficient for human needs.

2. "Land" is the whole of the material universe that has not undergone production.

3. "Labour" is human force applied to production.

4. "Production" (P), when used in a quantitative sense, refers to the value (referred to some convenient standard) of the products after they have undergone the process of production.

5. "Rent" (R) is that portion of production which is given up to landowners in return for benefits derived from land in their possession.

6. "Marginal production" (p) is the production which would be obtained if all the land were equal in productivity to the most productive land available without the payment of rent.

7. "Wages" (W) is that portion of production returned to labour in return for its co-operation in production.

8. "Capital" (C) is the surplus of production which is used to assist labour in further production, by means of costly appliances, &c.

9. "Interest" (I) is that portion of production which is delivered to capital as equal in value to the mean increase of raw products due to the vitality of nature.

10. "Rate of interest" is the fraction obtained by dividing interest by capital.

11. "Proportional profit" is the profit derived from a certain investment divided by the amount invested.

I.

If u be the proportional profit at one point and u' that at another where u is less than u', then motion will take place from u to u', because every one tries to make the greatest profit he can. Further, the greater the difference between u and u' the greater the velocity of adjustment. Therefore, if there be n proportional profits at n different points, there will be a tendency to motion which will cease when all the profits are equal. Therefore, if V, V$'$, V$''$, V$'''$, V$''''$, &c., be the amounts invested at different points, the condition that there should be equilibrium is that—

$$\frac{1}{V}\cdot\frac{dV}{dt}=\frac{1}{V'}\cdot\frac{dV'}{dt}=\frac{1}{V''}\cdot\frac{dV''}{dt}=\&c.$$

From this we may deduce a relation between property-values and rate of interest (r).

Let V = property-value.

$$r=\frac{1}{C}\cdot\frac{dC}{dt};$$

$$\text{but}\quad \frac{1}{V}\cdot\frac{dV}{dt}=\frac{1}{C}\cdot\frac{dC}{dt}=r.$$

Integrating, we get—

$$V = V_0 e^{rt},$$

$$\text{and}\quad C = C_0 e^{rt}.$$

This assumes that r is constant, and that all the rent is devoted to buying more land and the interest to increasing capital. Neither of these assumptions is true, for evidently a man who is both a landowner and a capitalist may be most erratic in his investments; but it seems evident that, since the area of land in use is limited, more rent will find its way to capital than interest to land, so that capital will increase more quickly than given above and land-values more slowly. We may, however, deduce a formula free from both these objections by replacing dV/dt by R; then V, R, and r are simultaneous values at any time, and therefore true for all time.

$$\frac{R}{V}=r.$$

Since R is always greater than 0, we see that when $r = 0$ $V = \infty$, and *vice versâ*. There is one case in which $R = 0$: that is at and below the margin of cultivation; the formula then gives $V = 0$. True, but of no importance.

II.

Rent is equal to production *minus* the marginal production.

Let productiveness mean the production from unit-area of ground, and let it be represented by y, and the marginal productiveness by g. Further, let $dR/dx = z$. Then we may write $y = g + z + f(x)$, so that the profit after the rent has been paid is $g + f(x)$; but at the margin no rent has to be paid, so that the profit there is g. Now, if equal areas of land be taken at the margin and at any other point, we have—

$$\frac{1}{C}\cdot\frac{dC}{dt}=\frac{1}{C'}\cdot\frac{dC'}{dt}$$

—here I use C and C′ to include not only capital, but also labour—

$$\therefore\quad \frac{g+f(x)}{C}=\frac{g}{C'};$$

$$\text{or}\quad (C-C')g=f(x)C'.$$

If the distribution of C be uniform, $f(x) = 0$, so that—

$$y = g + z.$$

Integrating between suitable limits,—

$$P = p + R,$$
$$\text{or} \quad R = P - p.$$

This is the ordinary theory of rent, which seems always to be deduced by placing the distribution of capital under a restriction; but this is more apparent than real, for C and C′ contain both capital and labour, and to put them equal only means that their joint effects are the same at all points, though the distribution of capital may be extremely variable. This agrees with observation. The less capital a man has to work his land the harder he has to work to keep afloat.

III.—Wages.

$$P = R + I + W,$$
$$= R + p,$$
$$\therefore \quad W = p - I = p - Cr.$$

This shows that if the capital increases whilst p remains constant the wages will fall, and that in new countries, where p is large and C small, the wages should be large.

We have seen in I. that when r is constant $C = C_0e^{rt}$. Now, suppose I to have reached a constant value, then $C = C_0 + It$. The corresponding land-value will be—

$$V = V_0e^{\frac{It}{c}}.$$

The Malthusian theory states that population is kept down by its pressure against production—that is, if n is the population and w the demand of each.

$$P = nw,$$
$$\therefore \quad I = p - P = -R,$$
$$\text{since} \quad nw = W.$$

Similarly, if the law were $p = nd$ we should get $I = 0$, which is equally untrue and absurd.

Let the coefficient of labour-saving devices (s) be measured by the production which can be done by unit labour when using a labour-saving device. Then $P = sN$, where N is the number of men required to do this production with this coefficient. If we put $R + I = mP$, where m is some proper fraction, we have—

$$P = mP + W = mP + nw;$$
$$\text{or} \quad P = \frac{nw}{1-m},$$

where n is the number of men available; so that—

$$w = \frac{N}{n}(1-m)s = \frac{P}{n}(1-m).$$

Since $R = P - p$, the effect of increasing P is to increase R without increasing the wages, as the latter are included in p. Therefore to increase w we must increase $(1-m)$—that is, decrease m. Or we may put it that since P is directly proportional to s we must increase the ratio of N/n, which may be done by either increasing N or decreasing n. Let us examine these ways more in detail.

The diminishing of n has in the past been the most common way of increasing wages, but it has been far from successful, having been brought about by wars, pestilences, &c., which tend to diminish P at the same time; also, the destruction of property and ruin of the country generally is so great that the increase of wages is negligible.

The increasing of N has been tried—relief-works, for example—but is expensive, wasteful, and not lasting, for as soon as the artificial stimulant is removed the wages must revert to their former state.

The decreasing of m is what the single-taxers aim at doing, and what the rating on unimproved values aims at effecting. We have seen in II. that rent is the natural outcome of variable productivity and cannot be done away with, but it might be collected by the Government and distributed in the form of efficacious and lasting public works.

ART. LIII.—*On the Phenomena of Variation and their Symbolic Expression.*

By E. G. BROWN.

[*Read before the Wellington Philosophical Society, 11th March, 1902.*]

Plates XXXVI., XXXVII.

"A PERSON who uses an imperfect theory with the confidence due only to a perfect one will naturally fall into abundance of mistakes; his predictions will be crossed by disturbing circumstances of which his theory is not able to take account, and his credit will be lowered by the failure. And inasmuch as more theories are imperfect than are perfect, and of those who attend to anything the number who acquire very sound habits of judging is small compared with that of those who do not get so far, it must have happened, as it has happened, that a great quantity of mistake has been made by those who do not understand the true use of an imperfect theory. Hence much discredit has been brought upon theory in general, and the schism of theoretical and practical men has arisen."—(DE MORGAN, "Penny Cyclopædia," Art. "Theory.")

INTRODUCTION.

The present writer proceeds upon the assumption that the means of comparing those theories which are used to predict