Art. XLV.—*Preferential Voting in Single-member Constituencies, with Special Reference to the Counting of Votes.*

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[*Read before the Wellington Philosophical Society, 24th September, 1913.*]

This subject was so fully discussed in October, 1882, by Professor E. J. Nanson, of Melbourne University, in a paper on "Methods of Election," read before the Royal Society of Victoria, that some apology seems to be necessary for discussing it again. But if an interval of thirty years is not in itself a sufficient reason for a restatement of the arguments then used, I may be pardoned for saying that, absolutely conclusive as Professor Nanson's arguments were, the method of counting the votes proposed by him was so cumbrous as to deter the ordinary politician from giving the case set forth in the paper the attention it deserved.

In this paper I have described a method of counting that appears to me much simpler than Nanson's, although in reality based on the same general principles as his; and I have added the outline of a proof which is, to some extent at all events, independent of his proof.

I shall begin by taking an example. Let us suppose that there are five candidates, of whom one is to be elected, and that there are 400 voting-papers, the votes being distributed as follows:—

Table I.—Order of Preference shown.

A B C D E	on 100	papers.
B A C E D	on 80	„
D E C A B	on 100	„
E D C B A	on 60	„
A C B E D	on 10	„
C B A D E	on 40	„
B C A D E	on 10	„

(I assume, for this example, that these are the only preferences shown out of all the possible permutations.) According to Nanson, the score-sheet would be then exhibited as follows:—

Table II.

—	First Choice.	Second Choice.	Third Choice.	Fourth Choice.	Fifth Choice.
A ..	110	80	50	100	60
B ..	90	140	10	60	100
C ..	40	20	340	0	0
D ..	100	60	0	150	90
E ..	60	100	0	90	150

In order to ascertain which should be rejected, Nanson constructs, after the method of Borda, the trial table below, which is obtained from Table II by multiplying the first choices by 4 (one less than the number of candidates), the second choices by 3, the third by 2, the fourth by 1, and the fifth by 0.

TABLE III (*First Trial Table*).

A	.. ..	440 + 240 + 100 + 100 + 0 = 880 points.
B	.. ..	360 + 420 + 20 + 60 + 0 = 860 „
C	.. ..	160 + 60 + 680 + 0 + 0 = 900 „
D	.. ..	400 + 180 + 0 + 150 + 0 = 730 „
E	.. ..	240 + 300 + 0 + 90 + 0 = 630 „

Trial Totals.

Nanson then rejects all those whose trial totals are not greater than 800, the average of all the trial totals. (He shows by a rigid process the soundness of this step.) A new score-sheet similar to II is then made for the remaining candidates, A, B, C, disregarding votes given for D and E. It will be as follows :—

TABLE IV.

—	First Choice.	Second Choice.	Third Choice.
A	110	180	110
B	90	200	110
C	200	20	180

Multiplying the first choices by 2, the second by 1, and the third by 0, we get a trial table similar to Table III, thus :—

TABLE V (*Second Trial Table*).

A ..	.. .. ..	220 + 180 + 0 = 400 points.
B ..	.. .. ..	180 + 200 + 0 = 380 „
C ..	.. .. ..	400 + 20 + 0 = 420 „

Trial Totals.

The average of the totals is 400 ; he cuts out A and B, who have totals not greater than the average, and C is accordingly elected. Assuming that the preferences are truly and rightly given, and that each voter exercises all his preferences, this method is infallible.

Nanson proposes to neglect the cases in which no preference is shown for more than a few candidates ; but his method could be made complete by distributing the points that would thus be unassigned in Table III equally among the unmarked candidates. It is generally agreed, however, that Nanson's method, although mathematically sound, is too cumbrous for actual use.

But it can be shown that a trial table constructed as Table VI below gives the same totals as Nanson's Trial Table III above, and that the other trial tables required can be derived directly from it without any fresh count or calculation, except mere addition of figures already obtained.

The Trial Table VI is thus constructed : We make as many lines and as many columns as there are candidates ; we count the number of ballot-papers on which the first candidate on the list is preferred to the second, all other names being disregarded, and papers on which no preference is shown for either of these candidates being divided equally between them. The number so found is inserted in the second line of the first column—that is, total preferences of A over B = 210. (The same count gives us the number for the first line of the second column—that is, the number of papers (190) on which the second candidate is preferred to the first.) Each of the other spaces in the table is similarly filled up.

TABLE VI *(First Trial Table)*

—	A.	B.	C.	D.	E.
Beats A ..	x	190	210	160	160
„ B ..	210	x	210	160	160
„ C ..	190	190	x	160	160
„ D ..	240	240	240	x	150
„ E ..	240	240	240	250	x
Trial totals..	880	860	900	730	630

D and E, *having trial totals not greater than the average, are rejected,* and the columns and lines for D and E are deleted.

The second trial table, which for clearness is here written out again, appears as follows :—

TABLE VII *(Second Trial Table).*

—	A.	B.	C.
Beats A	x	190	210
„ B	210	x	210
„ C	190	190	x
Trial totals ..	400	380	420

A's and B's trial totals being not greater than the average of the trial totals (400), they are rejected, and C is declared elected.

The soundness of this method can be proved independently of Nanson's method; for the successful candidate is, *generally*, the candidate who would be preferred to any other single candidate if they were the only two candidates: hence if n, which must be either of the form 2N or of the form 2N + 1, be the total number of voting-papers, the successful candidate must as against any other candidate obtain at least N + 1 votes, and his trial total must not be less than $(m - 1)$ (N + 1), where m is the number of candidates. These total preferences occur in pairs, and the sum of each pair is n—*e.g.*, in Table VI the total preferences of A over B is 210, and of B over A 190, and their sum is 400. The number of pairs is $\frac{m(m-1)}{1 \cdot 2}$; therefore the average of all the trial totals is $\frac{1}{2}nm\,(m - 1) \div m = \frac{1}{2}n\,(m - 1)$, which is either of the form $(m - 1)$ N, or of the form $(m - 1)$ (N + $\frac{1}{2}$). Any trial total not greater than this is less than $(m - 1)$ (N + 1). A candidate with a trial total not greater than the average of all the trial totals cannot, therefore, be elected, and may be thrown out at any stage.

I have said that *generally* the successful candidate is the candidate who would be preferred to any other single candidate if they were the only two candidates; but there are two other cases—namely, the case of equality, and what Nanson calls the inconsistent case. In each of these cases the same rule as that given above applies.

In the case of equality, when at the last stage two or more candidates are equal, and it is necessary to exercise a casting-vote, it is probably the best rule to give this to the candidate of the two or more equal candidates at the last count who stood highest at the first count; or, if this does not determine it, to the one who stood highest at the second count; or, if the candidates in question be equal at all counts, to determine the result by drawing lots.

In the "inconsistent case" there are, say, three candidates—A, B, C—of whom A beats B, B beats C, and C beats A. For instance, if the votes are distributed as in Table I, except that the last two lines are

C B A D E	26
B C A D E	24

then Table VI becomes

TABLE VIII (*First Trial Table*).

—	A.	B.	C.	D.	E.
Beats A ..	x	190	210	160	160
„ B ..	210	x	196	160	160
„ C ..	190	204	x	160	160
„ D ..	240	240	240	x	150
„ E ..	240	240	240	250	x
	880	884	886	730	630

and Table VII becomes

TABLE IX (*Second Trial Table*).

—	A.	B.	C.
Beats A	x	190	210
„ B	210	x	196
„ C	190	204	x
	400	394	406

Following the rule, we reject A and B, and C is elected. Is this result correct?

Now, Table IX shows that the majority that affirms that B is better than C (*i.e.*, 204) is less than the majority that affirms that C is better than A (namely, 210), and less than the majority that affirms that A is better than B (also 210); hence B should go out.

Again, C is better than A; therefore C is elected. (This proof is general in character.) It is evident that the candidate with the biggest net majority will get a greater trial total than the average of the trial totals, and therefore *all those with trial totals not greater than the average must be rejected,* which is the same rule as before. (Cases of equality may occur here also, and should be treated as before.)

It will not be necessary in many cases to fill up the whole of Table VI or Table VIII, which would require $\frac{1}{2}m\ (m - 1)$ entries in the trial table, where m is the number of candidates. (These counts in any case become simpler as the ballot-papers become sorted out in the process of counting.)

From the first preferences shown in Table I the first count would show —A, 110 votes; B, 90; C, 40; D, 100; E, 60. We could count first the papers on which A, the highest candidate in this count, beats C, and the number of papers on which C beats A. We find that C is preferred to A on 210 papers out of 400; A is therefore rejected (provisionally, at least), and his papers are distributed according to the second preferences shown on them. C is then matched against D, whom he beats by 240 to 160; against B, beating him by 210 to 190; and, finally, against E, obtaining 240 as against E's 160. C is therefore elected. Only $m - 1$ counts have been necessary (in this case, 4), and each count is simpler than the one before.

If in the above counting it had been found that B beat C, then it would have been necessary to try B against A to ascertain whether the inconsistent case had occurred; but even then the process of counting might be considerably shortened; and, in general, any candidate whose trial total has been completed may be rejected if that trial total is not greater than $\frac{1}{2}n\ (m - 1)$.

I have purposely chosen an example in which the lowest candidate on the first count is the candidate who should be declared elected, as it shows in a most striking manner the unsoundness of Ware's method, in which the lowest candidate of the first and each succeeding count is rejected. This is the method under the Electoral Act of 1907 in Western Australia. Still more fallacious is the Queensland method under the Act of 1905, where all the candidates except the two highest are rejected at the first count. The second ballot stands in the same category, and, moreover, is open to other objections, especially in that it requires two polls, and gives opportunity for intrigue of various kinds.

The method I have explained gives, I claim, the real choice of the electors, as far as that can be expressed by any system of voting, and it is not too complicated in operation. I venture to claim also that every elector could easily understand the principle involved in the counting: we simply cut out in turn each candidate that is shown to have no chance of election until we have one successful candidate left.

If proportional representation were to be finally adopted, an interim adoption of my method in single-member constituencies would train the electors in the habit of indicating their preferences

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