The Elasticity of the Earth's Central Core.

By K. E. Bullen, University of Melbourne.

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In a previous paper of the writer (1940), values were given for the various elastic constants of the Earth's outer mantle down to the boundary of the central core at a depth of 2900 km. below the Earth's surface. These values were published following a detailed examination (1940A) of the problem of the density distribution in the Earth's mantle. The writer (1941) has since completed a similar investigation for the Earth's central core, and has shown that within the greater part of the core, density values may be obtained within an uncertainty that in all probability does not exceed 3 per cent. An important factor in the attaining of this degree of precision has been the use of seismological tables of Jeffreys (1939), which have been derived as the conclusion of an extensive programme of work lasting several years and subjected throughout to thorough statistical treatment. With the use of Jeffreys' final values for the P velocity distribution, and of the writer's most recent density figures, it is now possible to determine fairly accurate figures for the elasticity of a large part of the central core.

It will be assumed here as in previous papers that rigidity is negligible throughout the central core. The seismological evidence for the existence of S waves in the core is very doubtful, and important evidence from other sources implies that even if the core is not actually fluid, its rigidity must be very small. We therefore neglect the rigidity μ, and so are left to determine only the one elastic constant k, the bulk modulus or incompressibility; k of course now becomes identical with the constant λ, and is determined by means of the relation

$$\alpha^2 = k/\rho \qquad (1)$$

where α is the velocity of seismic P waves, and ρ the density.

The region of the central core within which the density distribution is now known with close precision extends from the outer boundary of the core to a distance of 0·40 a from the Earth's centre, where a is the radius (approximately $3 \cdot 47 \times 10^8$ cm.) of the central core. This region of the Earth has been referred to in an earlier paper as layer E. The values of k within this layer E as calculated using equation (1) are given in Table I in terms of the depth d km. below the Earth's outer surface. The units in the table for k are 10^{12} dynes/cm^2, and having regard to the uncertainties in α and ρ, it follows that the values of k within layer E should be accurate within less than 4 per cent.

In spite of the large density jump from 5·68 to 9·43 gm./cm.^{3} which occurs at the boundary of the central core, it is rather interesting to note that the bulk modulus alters by only about 5 per cent. across this boundary, from 6·5 to $6 \cdot 2 \times 10^{12}$ dynes/cm^2. A much larger discontinuity in the value of k appears, however, to occur below layer E.

Figures for the density ρ and the pressure p as obtained in the density paper (1941) are also included in Table I, the units being gm./cm.^{3} and 10^{12} dynes/cm.^{2}, respectively. A further interesting feature is that k is seen to be nearly proportional to p within layer E, the value of the ratio k/p (which has zero dimensions) being also included in the table.

In Table II values of k are for convenience given in terms of the ratio r/a, where r is the distance from the Earth's centre.

Within a distance of 0·40 a (about 1400 km.) from the Earth's centre, the density results are subject to a much higher degree of uncertainty than is the case above this level. Immediately below layer E there appears to be a transition layer (F) of thickness about 0·04 a, within which the density increases at a sharply increased rate. At the bottom of layer F, there is a first order discontinuity in both velocity and density. From here to the Earth's centre, the velocity and density increase slowly and smoothly towards the Earth's centre within an inmost core, layer G, of radius about 0·36 a. The most probable value of the density of layer G. appears to be in the vicinity of 17 gm./cm.^{3} On the basis of this figure, it would follow that the value of k would range from $2{\cdot}1 \times 10^{13}$ dynes/cm^2 at the boundary of the layer to $2{\cdot}2 \times 10^{13}$ dynes/cm^2 at the Earth's centre. On account of the uncertainty in the density, however, these figures are uncertain within about $0{\cdot}5 \times 10^{13}$ dynes/cm^2 either way, so that at present only the order of magnitude of the compressibility can be reliably indicated for this region of the Earth. Within layer F the value of k changes continuously from the value $1{\cdot}26 \times 10^{13}$ dynes/cm^2 at the top of the layer, but there is no means of fixing accurately the rate of change. Thus for this layer also, only the order of the magnitude of the compressibility is indicated.

The two layers F and G together, however, occupy only one-hundredth of the Earth's total volume. Thus the results in the following tables together with those in an earlier paper actually fix with good precision the elastic constants for 99 per cent. (by volume) of the Earth's material. In addition, the results of the present paper indicate the order of magnitude of the compressibility in the remaining 1 per cent.

TABLE I.

d	ρ	k	p	k/p
2900	9.43	6.19	1.37	4.5
3000	9.57	6.47	1.47	4.4
3200	9.85	7.07	1.67	4.2
3400	10.11	7.75	1.85	4.2
3600	10.35	8.44	2.04	4.1
3800	10.56	9.09	2.22	4.1
4000	10.76	9.71	2.40	4.1
4200	10.94	10.29	2.57	4.0
4400	11.11	10.85	2.73	4.0
4600	11.27	11.42	2.88	4.0
4800	11.41	12.00	3.03	4.0
4982	11.54	12.58	3 17	4.0

TABLE II.

d	r/a	k
2900	1.0	6.19
3074	0.95	6.68
3247	0.90	7.22
3421	0.85	7.82
3594	0.80	8.43
3768	0.75	8.99
3941	0.70	9.54
4115	0.65	10.05
4288	0.60	10.54
4462	0.55	11.02
4635	0.50	11.52
4809	0.45	12.04
4982	0.40	12.58

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