A Table of Debye-Hückel Functions for Use in Solution Chemistry.

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SINCE the appearance of the theory of Debye and Hückel (1923), the idea of interionic attraction, postulated by these authors, has been generally accepted as representing one important factor describing the deviation of electrolytic solutions from the laws of ideal or perfect solutions. Debye and Hückel derived an equation for the activity coefficient of an electrolyte which, for an aqueous solution at a temperature of 25°, takes the form:

$$-\log f = 0{\cdot}5066\ z_1 z_2 \sqrt{I}/(1+0{\cdot}3288 a \sqrt{I}) \qquad (1)$$

where f is the rational activity coefficient, z_1 and z_2 are the valences of the ions of the electrolyte, I is the total ionic strength and a is a constant, characteristic of the electrolyte, designated as the distance of closest approach of the ions. The total ionic strength, I, is defined as $\frac{1}{2}\sum c_i z_i^2$, where the summation is taken over all the kinds of ions present in solution and c_i denotes ionic concentration in gram-ions per litre. Thus for 1–1, 1–2, 2–2 and 1–3 electrolytes $I = c$, $3c$, $4c$ and $6c$ respectively, c being the electrolyte concentration in mols per litre.

Equation (1) is of great importance in the theory of solutions and leads to a large amount of computation when values of log f are required over a range of c and a values. We have now constructed a table which facilitates these computations. The table is based on the following considerations. Rearranging equation (1) we obtain:

$$-z_1 z_2/\log f = (1{\cdot}974/\sqrt{I}) + 0{\cdot}649\ a \qquad (2)$$

The table at the end of this paper contains, in the two right-hand columns, values of $0{\cdot}649\ a$ corresponding to a range of a values. The main body of the table contains values of $1{\cdot}974/\sqrt{I}$ for a range of I values compiled, like a table of logarithms, so that values can be interpolated corresponding to four significant figures in I. It is important to note that the *figures in the difference columns on the right are to be subtracted.* The negative logarithm of the activity coefficient, —log f, is obtained for any value of a and c by finding the two values of $1{\cdot}974/\sqrt{I}$ and $0{\cdot}649\ a$ from the table; the two values are added together and the reciprocal obtained from a table of reciprocals. This reciprocal is multiplied by the valence product, $z_1 z_2$, to give —log f.

The following examples will illustrate the use of the table as well as the extension to concentrations outside the range listed in the table:—

Valence Type.	Example.	z_1z_2.	c.	I.	$\mathring{a}$	$1.974/\sqrt{I}$.	$0.649\,\mathring{a}$	Sum.	Reciprocal.	— log f.
1–1	KCl	1	0.2987	0.2987	5.0	3.612	3.245	6.857	0.1458	0.1458
1–1		1	0.002987	0.002987	5.0	36.12	3.245	39.365	0.0254	0.0254
1–1		1	0.02987	0.02987	5.0	11.42	3.245	14.665	0.0682	0.0682
1–1		1	0.02987	0.02987	4.0	11.42	2.596	14.016	0.0713	0.0713
1–2	$BaCl_2$	2	0.5	1.5	4.0	1.612	2.596	4.208	0.2377	0.4754
1–3	$LaCl_3$	3	0.00073	0.00438	5.0	29.83	3.245	33.075	0.0302	0.0906
2–2	$ZnSO_4$	4	0.5	2.0	5.0	1.396	3.245	4.641	0.2155	0.8620

Reference.

Debye, P., and Hückel, E., 1923. *Physikal. Z.*, *24*, 185.

TABLE OF DEBYE–HÜCKEL FUNCTIONS.

I	0	1	2	3	4	5	6	7	8	9	Differences (to be subtracted, not added). 1	2	3	4	5	6	7	8	9
0.10	6.243	6.212	6.181	6.151	6.121	6.092	6.063	6.035	6.007	5.979	3	6	9	12	15	18	21	24	26
.11	5.952	5.925	5.899	5.873	5.847	5.821	5.796	5.771	5.747	5.723	3	5	8	10	13	15	18	20	23
.12	5.699	5.675	5.652	5.629	5.606	5.584	5.561	5.539	5.518	5.496	2	5	7	9	11	14	16	18	20
.13	5.475	5.454	5.434	5.413	5.393	5.373	5.353	5.333	5.314	5.295	2	4	6	8	10	12	14	16	18
.14	5.276	5.257	5.239	5.220	5.202	5.184	5.166	5.149	5.131	5.114	2	4	5	7	9	11	13	14	16
.15	5.097	5.080	5.064	5.047	5.030	5.014	4.998	4.982	4.966	4.951	2	3	5	7	8	10	11	13	15
.16	4.935	4.920	4.905	4.890	4.875	4.860	4.845	4.831	4.816	4.802	1	3	4	6	7	9	10	12	13
.17	4.788	4.774	4.760	4.746	4.733	4.719	4.706	4.692	4.679	4.666	1	3	4	5	7	8	9	11	12
.18	4.653	4.640	4.627	4.615	4.602	4.590	4.577	4.565	4.553	4.541	1	3	4	5	6	8	9	10	11
.19	4.529	4.517	4.505	4.494	4.482	4.470	4.459	4.448	4.436	4.425	1	2	3	5	6	7	8	9	10
.20	4.414	4.403	4.392	4.382	4.371	4.360	4.349	4.339	4.329	4.318	1	2	3	4	5	6	8	9	10
.21	4.308	4.298	4.288	4.277	4.267	4.257	4.248	4.238	4.228	4.218	1	2	3	4	5	6	7	8	9
.22	4.209	4.199	4.190	4.180	4.171	4.162	4.153	4.143	4.134	4.125	1	2	3	4	5	6	7	7	8
.23	4.116	4.107	4.099	4.090	4.081	4.072	4.064	4.055	4.047	4.038	1	2	3	3	4	5	6	7	8
.24	4.030	4.021	4.013	4.005	3.996	3.988	3.980	3.972	3.964	3.956	1	2	2	3	4	5	6	7	7
.25	3.948	3.940	3.932	3.925	3.917	3.909	3.902	3.894	3.886	3.879	1	2	2	3	4	5	5	6	7
.26	3.872	3.864	3.857	3.849	3.842	3.835	3.828	3.820	3.813	3.806	1	1	2	3	4	4	5	6	7
.27	3.799	3.792	3.785	3.778	3.771	3.764	3.758	3.751	3.744	3.737	1	1	2	3	3	4	5	6	6
.28	3.731	3.724	3.717	3.711	3.704	3.698	3.691	3.685	3.678	3.672	1	1	2	3	3	4	5	5	6
.29	3.666	3.660	3.653	3.647	3.641	3.635	3.628	3.622	3.616	3.610	1	1	2	2	3	4	4	5	6
.30	3.604	3.598	3.592	3.586	3.580	3.575	3.569	3.563	3.557	3.551	1	1	2	2	3	4	4	5	5
.31	3.546	3.540	3.534	3.529	3.523	3.517	3.512	3.506	3.501	3.495	1	1	2	2	3	3	4	4	5
.32	3.490	3.484	3.479	3.474	3.468	3.463	3.458	3.452	3.447	3.442	1	1	2	2	3	3	4	4	5
.33	3.436	3.431	3.426	3.421	3.416	3.411	3.406	3.401	3.396	3.391	1	1	2	2	3	3	4	4	5
.34	3.386	3.381	3.376	3.371	3.366	3.361	3.356	3.351	3.346	3.342	0	1	1	2	2	3	3	4	4
.35	3.337	3.332	3.327	3.323	3.318	3.313	3.309	3.304	3.299	3.295	0	1	1	2	2	3	3	4	4
.36	3.290	3.286	3.281	3.277	3.272	3.268	3.263	3.259	3.254	3.250	0	1	1	2	2	3	3	4	4
.37	3.245	3.241	3.237	3.232	3.228	3.224	3.219	3.215	3.211	3.207	0	1	1	2	2	3	3	3	4
.38	3.202	3.198	3.194	3.190	3.186	3.182	3.177	3.173	3.169	3.165	0	1	1	2	2	2	3	3	4
.39	3.161	3.157	3.153	3.149	3.145	3.141	3.137	3.133	3.129	3.125	0	1	1	2	2	2	3	3	4
.40	3.121	3.117	3.114	3.110	3.106	3.102	3.098	3.094	3.091	3.087	0	1	1	2	2	2	3	3	3
.41	3.083	3.079	3.076	3.072	3.068	3.064	3.061	3.057	3.053	3.050	0	1	1	1	2	2	3	3	3
.42	3.046	3.043	3.039	3.035	3.032	3.028	3.025	3.021	3.017	3.014	0	1	1	1	2	2	2	3	3
.43	3.011	3.007	3.003	3.000	2.997	2.993	2.990	2.986	2.983	2.979	0	1	1	1	2	2	2	3	3
.44	2.976	2.973	2.969	2.966	2.963	2.959	2.956	2.953	2.949	2.946	0	1	1	1	2	2	2	3	3
.45	2.943	2.940	2.936	2.933	2.930	2.927	2.923	2.920	2.917	2.914	0	1	1	1	2	2	2	3	3
.46	2.911	2.907	2.904	2.901	2.898	2.895	2.892	2.889	2.886	2.883	0	1	1	1	2	2	2	2	3
.47	2.880	2.877	2.873	2.870	2.867	2.864	2.861	2.858	2.855	2.852	0	1	1	1	2	2	2	2	3
.48	2.849	2.846	2.844	2.841	2.838	2.835	2.832	2.829	2.826	2.823	0	1	1	1	1	2	2	2	3
.49	2.820	2.817	2.814	2.812	2.809	2.806	2.803	2.800	2.797	2.795	0	1	1	1	1	2	2	2	3
.50	2.792	2.789	2.786	2.783	2.781	2.778	2.775	2.772	2.770	2.767	0	1	1	1	1	2	2	2	2
.51	2.764	2.762	2.759	2.756	2.754	2.751	2.748	2.746	2.743	2.740	0	1	1	1	1	2	2	2	2
.52	2.738	2.735	2.732	2.730	2.727	2.725	2.722	2.719	2.717	2.714	0	1	1	1	1	2	2	2	2
.53	2.712	2.709	2.707	2.704	2.701	2.699	2.696	2.694	2.691	2.689	0	1	1	1	1	2	2	2	2
.54	2.686	2.684	2.681	2.679	2.677	2.674	2.672	2.669	2.667	2.664	0	0	1	1	1	1	2	2	2
.55	2.662	2.659	2.657	2.655	2.652	2.650	2.647	2.645	2.643	2.640	0	0	1	1	1	1	2	2	2
.56	2.638	2.636	2.633	2.631	2.629	2.626	2.624	2.622	2.619	2.617	0	0	1	1	1	1	2	2	2
.57	2.615	2.613	2.610	2.608	2.606	2.603	2.601	2.599	2.597	2.594	0	0	1	1	1	1	2	2	2
.58	2.592	2.590	2.588	2.585	2.583	2.581	2.579	2.577	2.574	2.572	0	0	1	1	1	1	2	2	2
.59	2.570	2.568	2.566	2.564	2.561	2.559	2.557	2.555	2.553	2.551	0	0	1	1	1	1	1	2	2
0.6	2.549	2.528	2.507	2.487	2.468	2.449	2.430	2.412	2.394	2.377	2	4	6	8	10	11	13	15	17
0.7	2.359	2.343	2.327	2.311	2.295	2.280	2.265	2.250	2.235	2.221	2	3	5	6	8	9	11	12	14
0.8	2.207	2.193	2.180	2.167	2.154	2.141	2.129	2.116	2.104	2.093	1	3	4	5	6	8	9	10	12
0.9	2.081	2.069	2.058	2.047	2.036	2.025	2.015	2.004	1.994	1.984	1	2	3	4	5	6	8	9	10
1.0	1.974	1.964	1.955	1.945	1.936	1.927	1.917	1.908	1.900	1.891	1	2	3	4	5	6	7	7	8
1.1	1.882	1.874	1.865	1.857	1.849	1.841	1.833	1.825	1.817	1.810	1	2	2	3	4	5	6	6	7
1.2	1.802	1.795	1.787	1.780	1.773	1.766	1.759	1.752	1.745	1.738	1	1	2	3	4	4	5	6	6
1.3	1.731	1.725	1.718	1.712	1.705	1.699	1.693	1.687	1.681	1.674	1	1	2	3	3	4	4	5	6
1.4	1.668	1.663	1.657	1.651	1.645	1.639	1.634	1.628	1.623	1.617	1	1	2	2	3	3	4	5	5
1.5	1.612	1.607	1.601	1.596	1.591	1.586	1.581	1.576	1.571	1.566	1	1	2	2	3	3	4	4	5
1.6	1.561	1.556	1.551	1.546	1.542	1.537	1.532	1.528	1.523	1.519	0	1	1	2	2	3	3	4	4
1.7	1.514	1.510	1.505	1.501	1.497	1.492	1.488	1.484	1.480	1.476	0	1	1	2	2	3	3	3	4
1.8	1.471	1.467	1.463	1.459	1.455	1.451	1.448	1.444	1.440	1.436	0	1	1	2	2	2	3	3	4
1.9	1.432	1.428	1.425	1.421	1.417	1.414	1.410	1.407	1.403	1.399	0	1	1	1	2	2	3	3	3
2.0	1.396	1.392	1.389	1.386	1.382	1.379	1.375	1.372	1.369	1.366	0	1	1	1	2	2	2	3	3
2.1	1.362	1.359	1.356	1.353	1.350	1.346	1.343	1.340	1.337	1.334	0	1	1	1	2	2	2	3	3
2.2	1.331	1.328	1.325	1.322	1.319	1.316	1.313	1.310	1.307	1.305	0	1	1	1	1	2	2	2	3
2.3	1.302	1.299	1.296	1.293	1.291	1.288	1.285	1.282	1.280	1.277	0	1	1	1	1	2	2	2	2
2.4	1.274	1.272	1.269	1.266	1.264	1.261	1.259	1.256	1.254	1.251	0	1	1	1	1	2	2	2	2
2.5	1.249	1.246	1.244	1.241	1.239	1.236	1.234	1.231	1.229	1.227	0	0	1	1	1	1	2	2	2
2.6	1.224	1.222	1.220	1.217	1.215	1.213	1.210	1.208	1.206	1 204	0	0	1	1	1	1	2	2	2
2.7	1.201	1.199	1.197	1.195	1.193	1.190	1.188	1.186	1.184	1.182	0	0	1	1	1	1	2	2	2
2.8	1.180	1.178	1.176	1.174	1.171	1.169	1.167	1.165	1.163	1.161	0	0	1	1	1	1	1	2	2
2.9	1.159	1.157	1.155	1.153	1.151	1.149	1.147	1.146	1.144	1.142	0	0	1	1	1	1	1	2	2
3	1.140	1.121	1.104	1.087	1.071	1.055	1.040	1.026	1.013	1.000	2	3	5	6	8	10	11	13	14
4	0.987	0.975	0.963	0.952	0.941	0.931	0.921	0.911	0.901	0.892	1	2	3	4	5	6	8	9	10
5	0.882	0.874	0.866	0.858	0.850	0.842	0.834	0.827	0.820	0.813	1	2	2	3	4	5	5	6	7
6	0.806	0.799	0.793	0.787	0.780	0.774	0.768	0.763	0.757	0.752	1	1	2	2	3	4	4	5	5
7	0.746	0.741	0.736	0.731	0.726	0.721	0.716	0.711	0.707	0.702	0	1	1	2	2	3	3	4	4
8	0.698	0.694	0.689	0.685	0.681	0.677	0.673	0.669	0.665	0.662	0	1	1	2	2	2	3	3	4
9	0.658	0.654	0.651	0.647	0.644	0.640	0.637	0.634	0.631	0.627	0	1	1	1	2	2	2	3	3

Å	0.649 Å
2.0	1.298
2.1	1.363
2.2	1.428
2.3	1.493
2.4	1.558
2.5	1.623
2.6	1.688
2.7	1.752
2.8	1.817
2.9	1.882
3.0	1.947
3.1	2.012
3.2	2.077
3.3	2.142
3.4	2.207
3.5	2.272
3.6	2.337
3.7	2.402
3.8	2.466
3.9	2.531
4.0	2.596
4.1	2.661
4.2	2.726
4.3	2.791
4.4	2.856
4.5	2.921
4.6	2.986
4.7	3.051
4.8	3.116
4.9	3.180
5.0	3.245
5.1	3.310
5.2	3.375
5.3	3.440
5.4	3.505
5.5	3.570
5.6	3.635
5.7	3.700
5.8	3.765
5.9	3.830
6.0	3.894
6.1	3.959
6.2	4.024
6.3	4.089
6.4	4.154
6.5	4.219
6.6	4.284
6.7	4.349
6.8	4.414
6.9	4.479
7.0	4.543